Photographic Technology

PhotoTechEDU series

Lecture 3: January 31, 2007
Ray Tracing: lenses & mirrors

Rom Clement

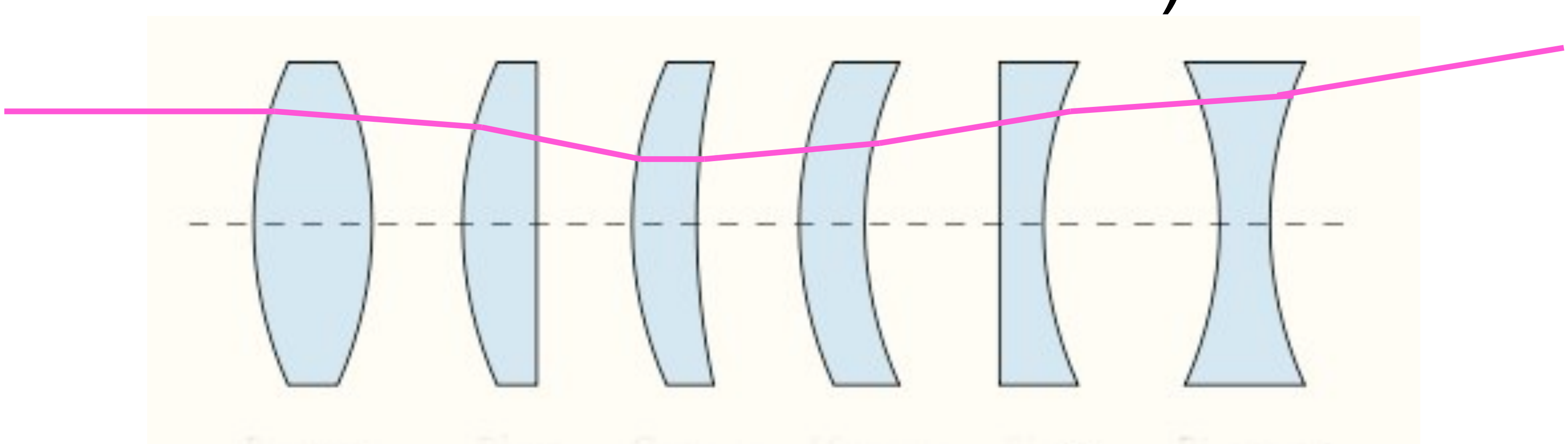
Google

Overview

- Description of lenses and mirrors
- Ray tracing
  - Principle of lenses and mirrors
  - Notion of real/virtual object & real/virtual image
  - Assumptions for geometric ray tracing (stigmatism)
  - Ray tracing for lenses and mirrors
- More advanced notions: thick lenses
  - Principal planes/points
  - Nodal points
- Quick overview of optical devices (if time allows it!)
  - The eye
  - Refractive and reflective telescopes

Today's lecture goal

- Given a set of lenses and/or mirrors
  - Trace essential rays
  - Compute the position of the image
  - Compute the equivalent focal length…
- …but we won't study the details of refraction inside a lens (or the details of reflection on mirrors surfaces)



Lenses & mirrors
Principles

Reflection & refraction

Reflection/Refraction

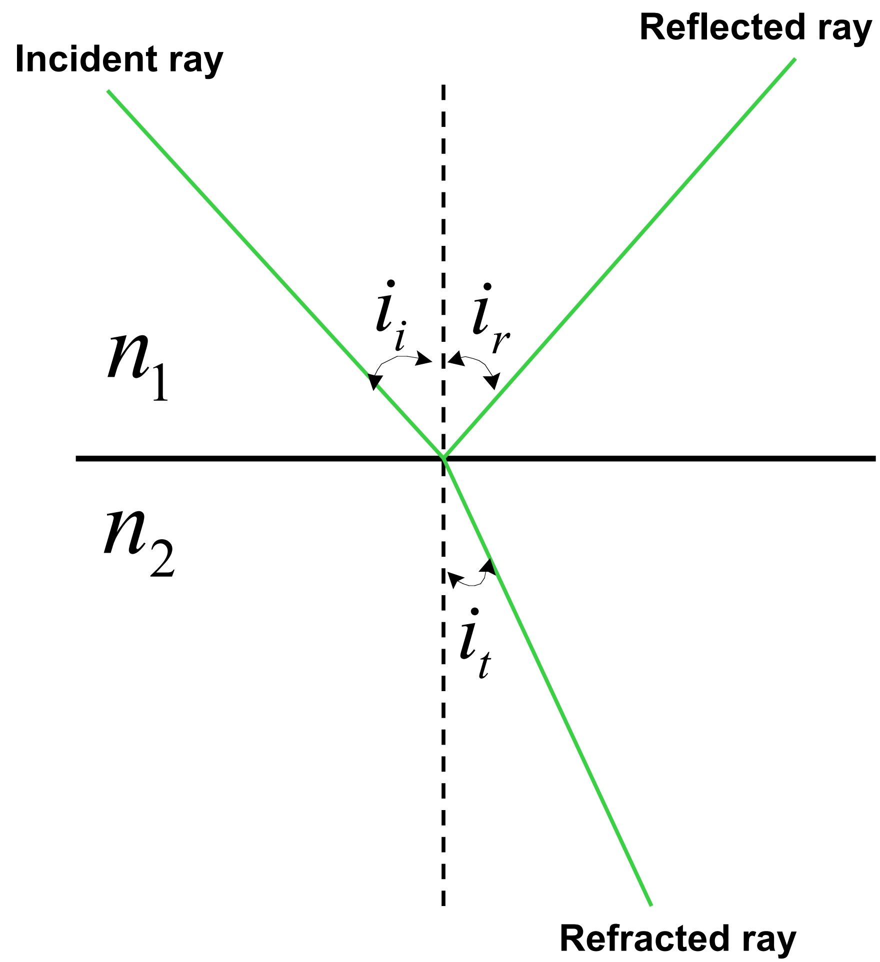
- Reflection law

$$i_i = i_r$$

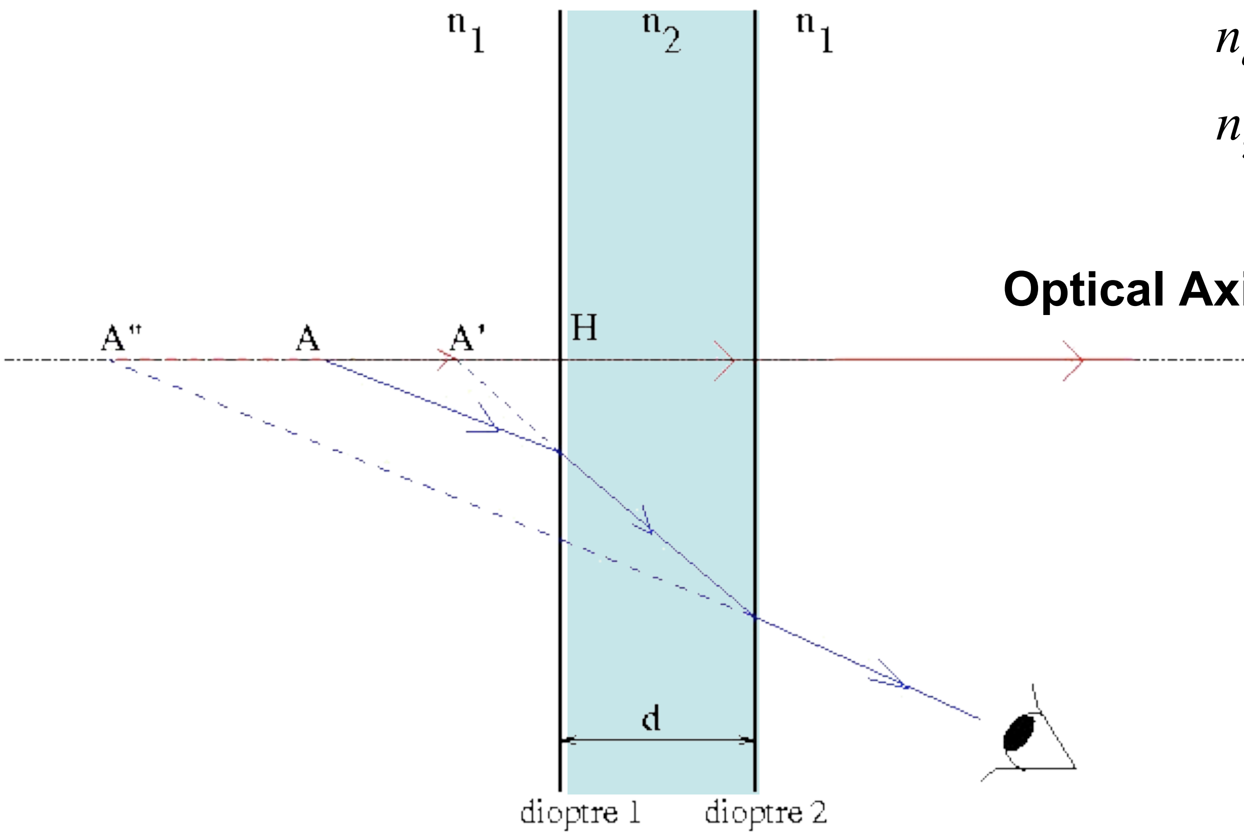
- Snell's refraction law

$$n_1 \sin(i_i) = n_2 \sin(i_t)$$

- Reflection is used with mirrors
- Refraction is used with lenses



Principles of lenses



$$n_{air} = 1.0002926$$

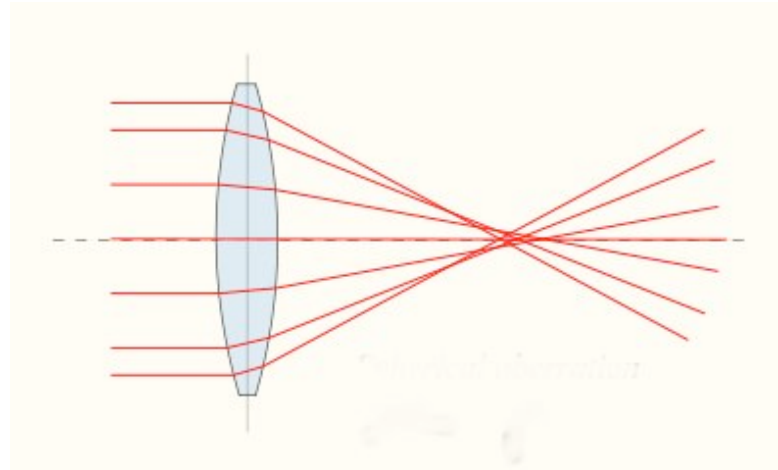
$$n_{\text{glass}}(\lambda) = 1.45 - 2$$

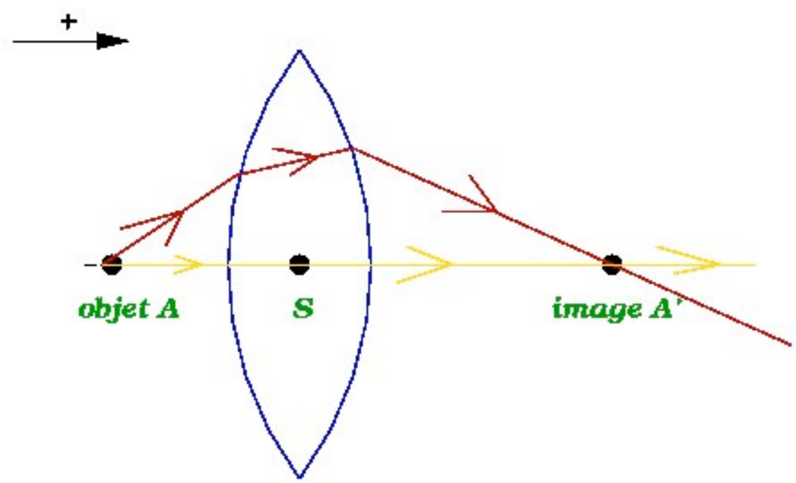
Optical Axis

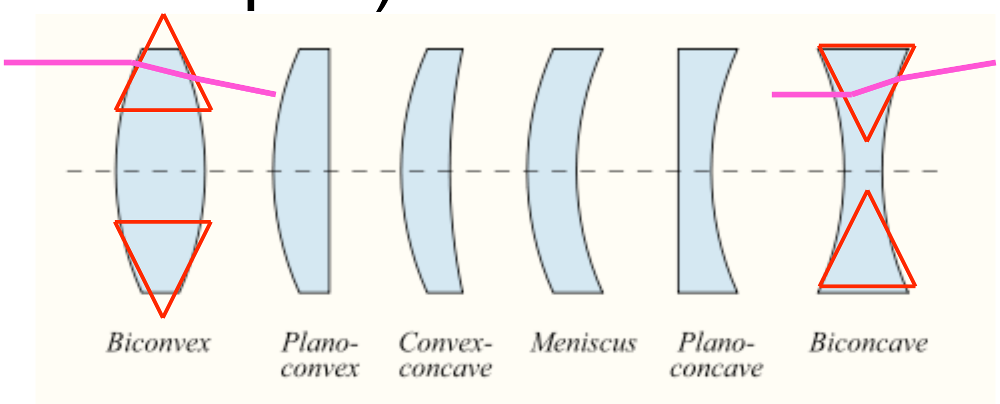
A lens is the association of 1 or 2 curved surfaces

Description of a lens

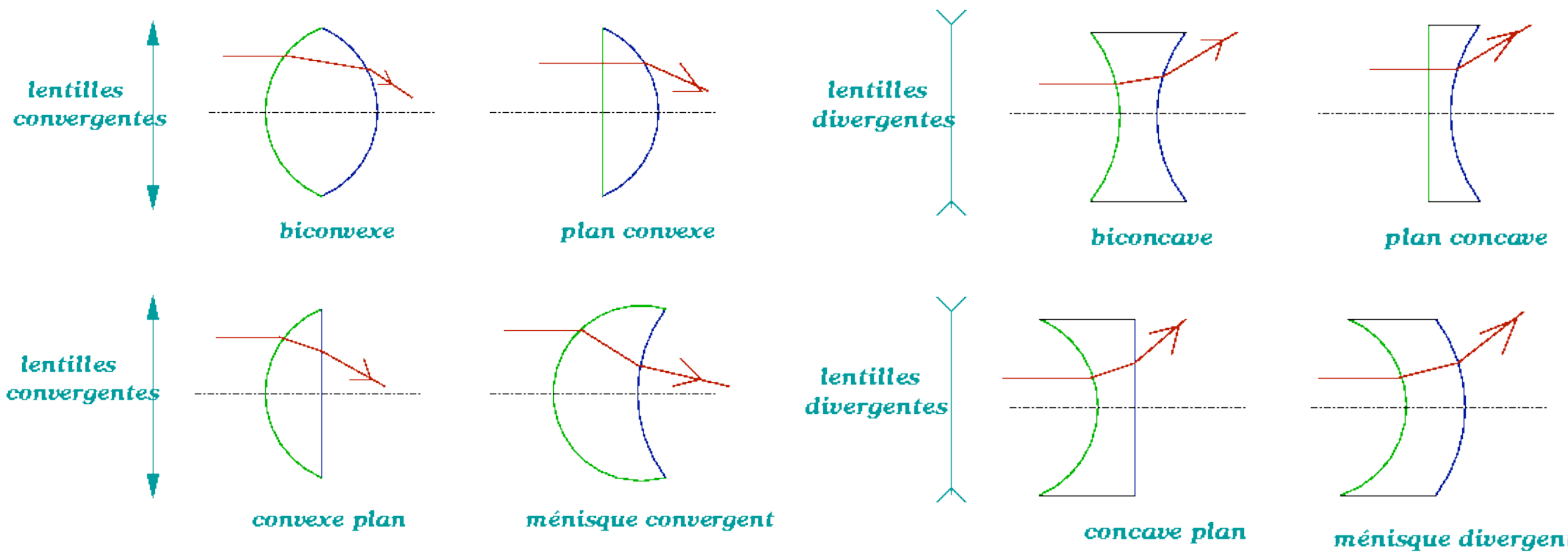
- Cross section reveals "prismatic" shape of lenses
- The total deviation is the sum of the 2 refractive deviations (on each diopter)







Overview of lenses

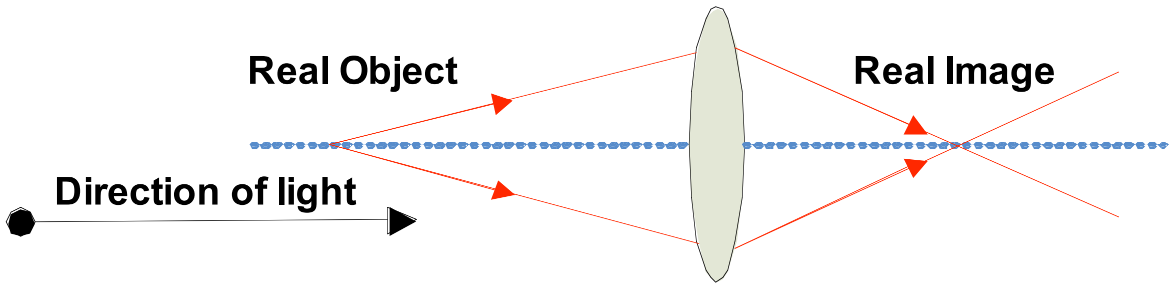


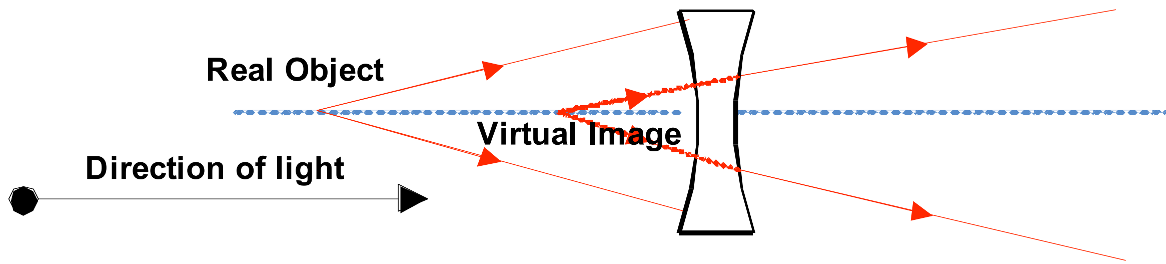
2 categories

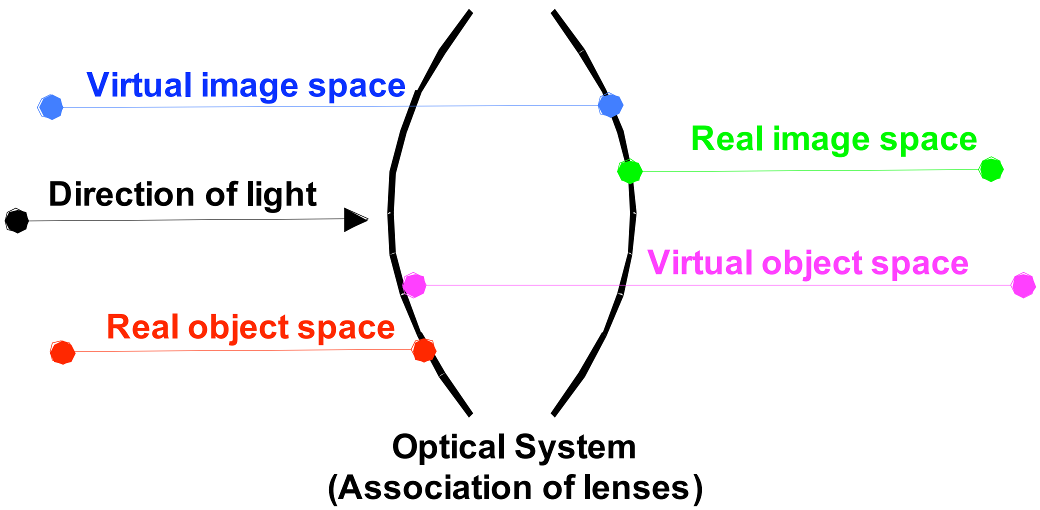
- **Thin edges (biconvex, plane convex, converging meniscus)**
- **Thick edges (biconcave, plane concave, diverging meniscus)**

Could do the study at each diopter level… Not the goal of this course

Lenses: real/virtual object & real/virtual image

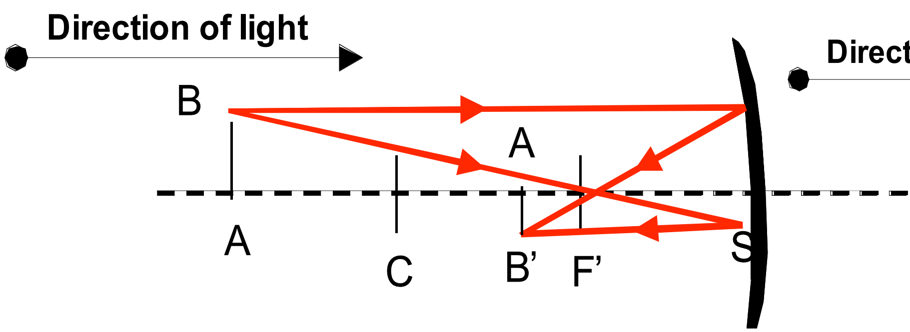




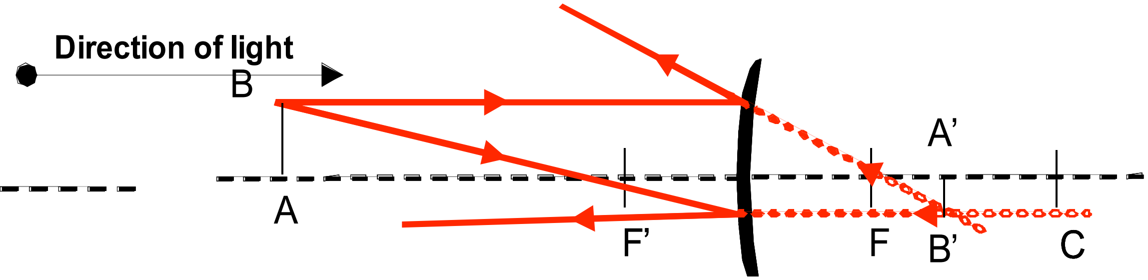


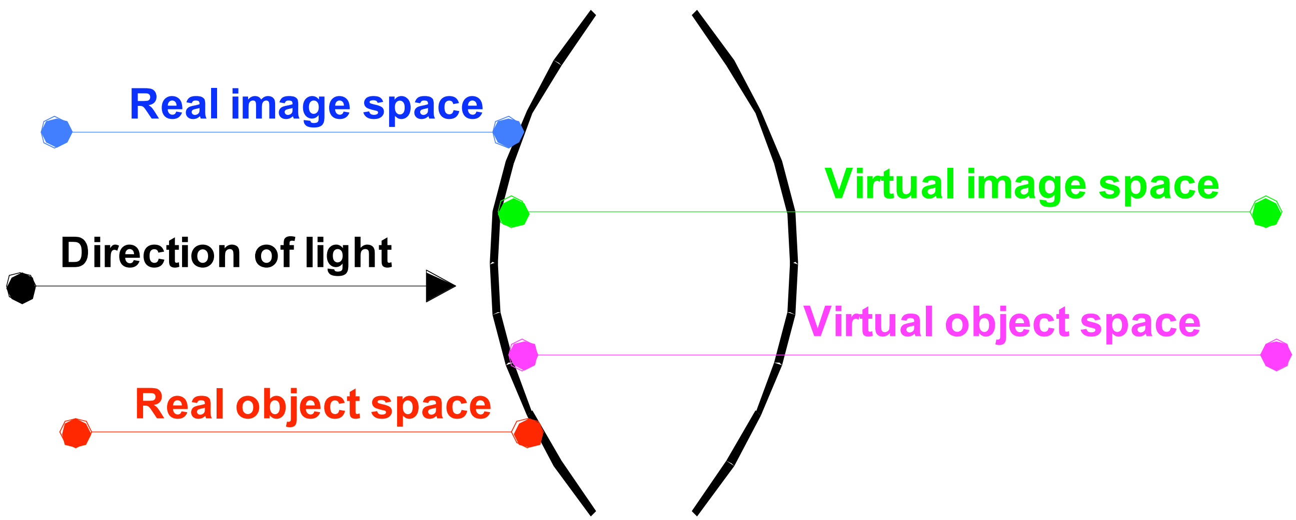
Mirrors: real/virtual object & real/virtual image

Concave Mirror



Convex Mirror





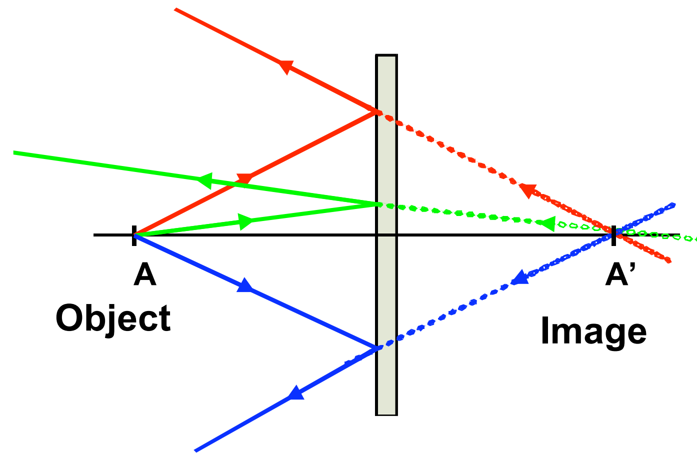
Stigmatism

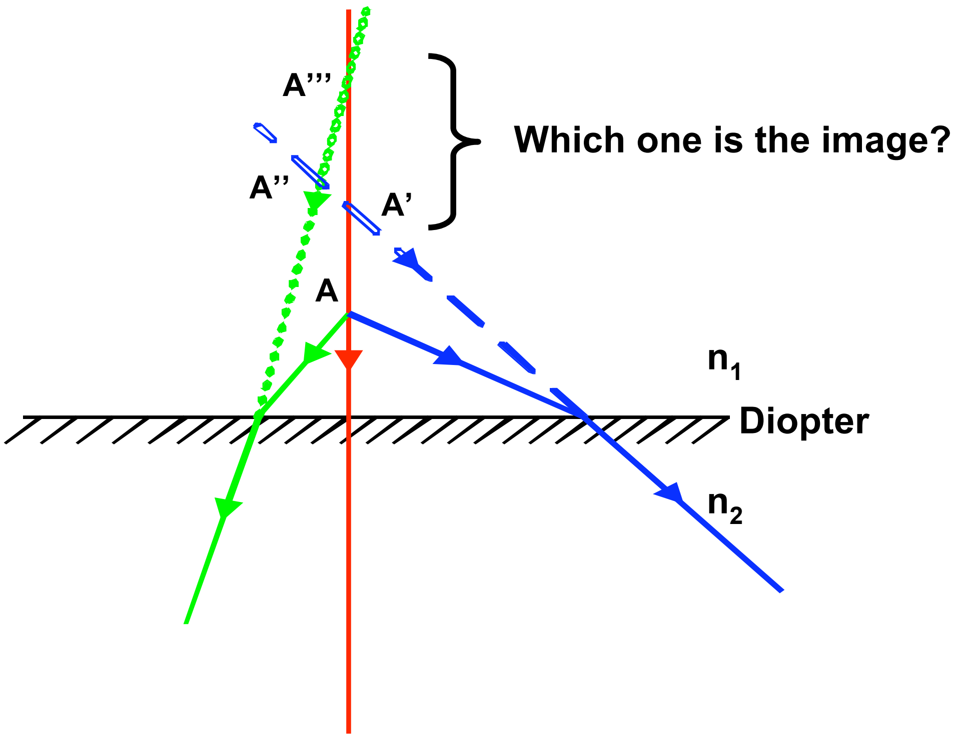
Assumptions for geometric ray tracing

Rigorous (and non rigorous) stigmatism

An optical system is rigorously stigmastic for a couple of points A, A’ if all rays passing through the point A go through the point A’

It is the case for a plane mirror

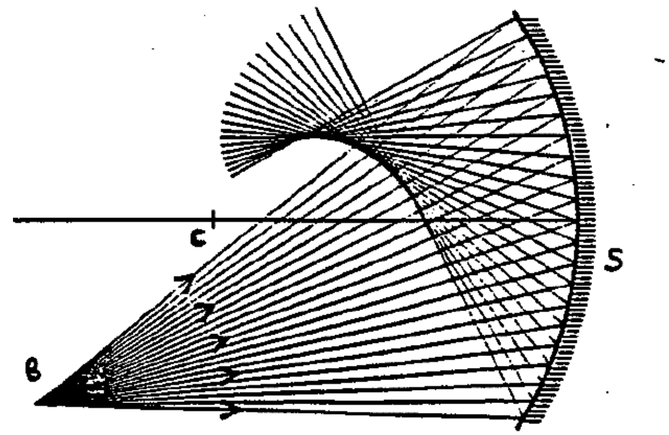


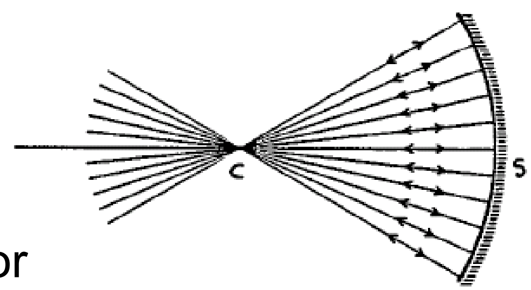


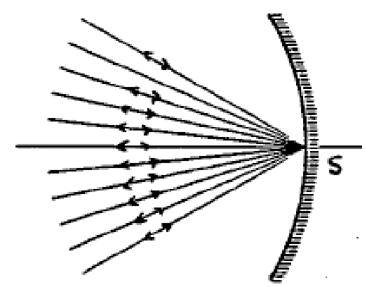
But even not for a simple diopter !

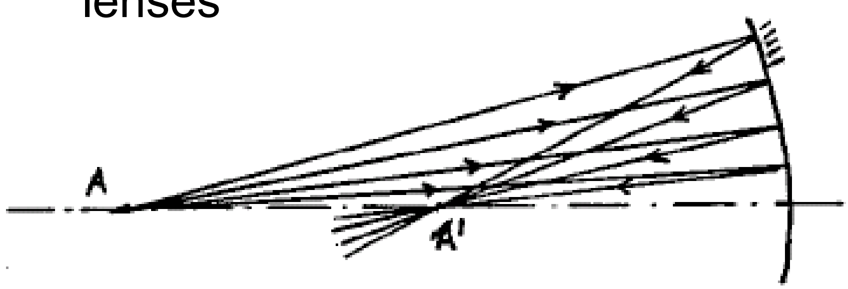
Non rigorous stigmatism and approximated stigmatism

- No rigorous stigmatism
  - There no rigorous stigmatism by nature with spherical surfaces/diopters…
  - Except for the mirrors at the curvature center (point C) or at the summit of the center (point S)
- But under gauss assumptions, the stigmatism is relatively good (in relation with the sensor spatial resolution)
  - It is true for mirrors as well as for lenses









Conditions of approximations for geometric ray tracing

- Gauss’s conditions for centered systems
  - Incident beam is centered at the center of the optical components (if not, use a diaphragm/pupil!)
  - The angle of the incident beam is “small”
- Consequences
  - Stigmatism is well approximated (in relation with the sensor spatial resolution)
  - Thin lenses has ~ symmetrical behavior
  - Things are simple and manageable by hand!

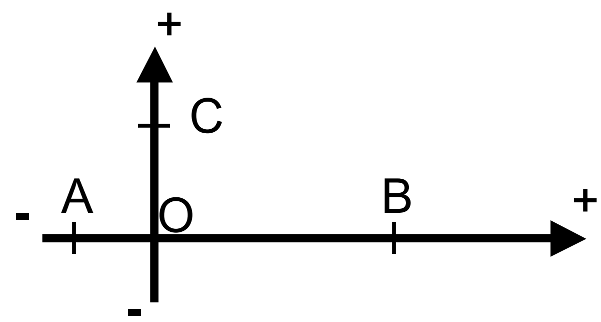
Note: a diaphragm does not reduce the field of view, it just reduces the diameter of the incoming beam of light. But a diaphragm reduces the amount of incident light, which induces a loss of luminosity and therefore may require an increase of exposure time…

Ray tracing
Lenses & Mirrors

Algebraic distances

Will use “oriented” distances

- Formulae valid whatever the curvature of the optical component is
- Allow to position faster the image
- It’s like vectors but we can divide by an algebraic distance



$$\overline{\mathrm{OB}} > \overline{\mathrm{AO}} > 0 > \overline{\mathrm{OA}} > \overline{\mathrm{BO}}$$

$$\overline{\mathrm{OB}} + \overline{\mathrm{BA}} = \overline{OA}$$

$$\overline{OC} > 0 > \overline{CO}$$

$$\frac{\overline{CO}}{\overline{AO}} < 0$$

$$\left|\frac{\overline{CO}}{\overline{AO}}\right| > 1$$

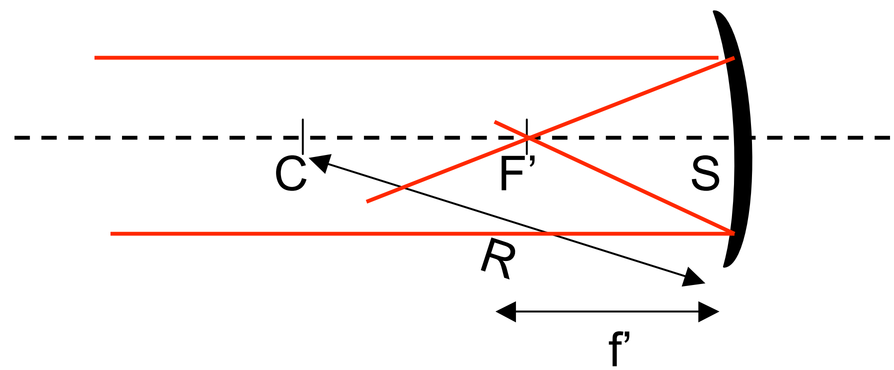
Nomenclature

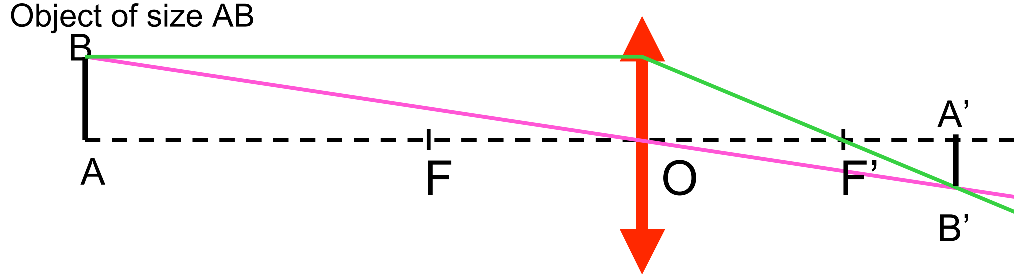
- **The light goes from the LEFT to the RIGHT**

- C = center of curvature of the mirror
- O = center of the lens = point so that all incident beam going through it is not deviated
- S = summit of the mirror (located on optical axis)
- F = object foyer = (also called first focal point) = point of the optical axis whose image is located at infinity on the optical axis
- F' = image foyer (also called second focal point) = image of a punctual object located at infinity on the optical axis
- A,B represents the object, A',B' is the image of the points A and B respectively

- $\gamma = \frac{\overline{A'}\overline{B'}}{\overline{A}\overline{B}}$ Magnification, ratio of the image's size/object's size

- Under Gauss' conditions
  - OF' = FO = f' = focal length of lens
  - CF'=CF = f' = focal length of mirror (F = F')
  - F'S = f' = CS/2

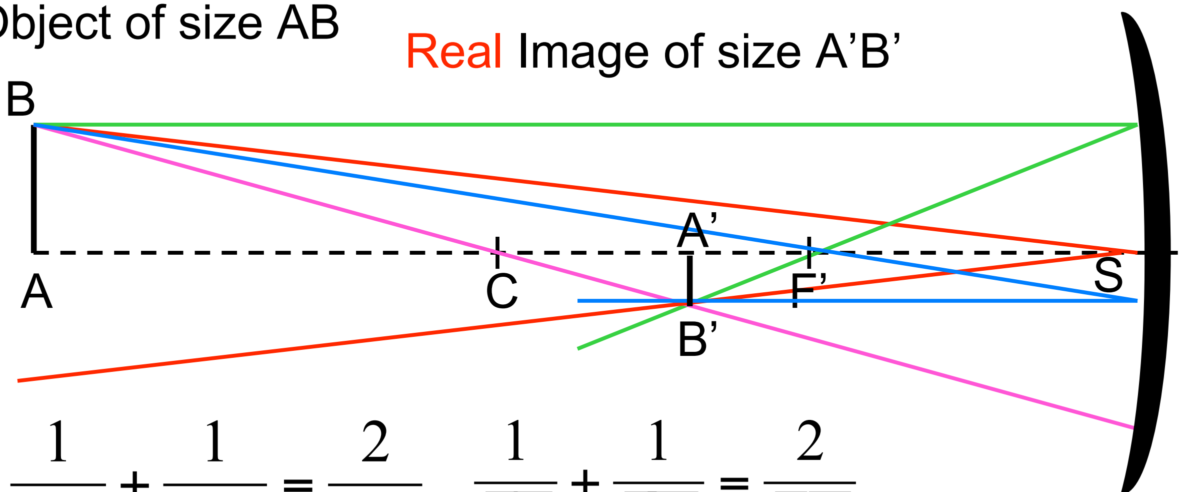




Ray tracing: case of a concave mirror

Real Object of size AB

Real Image of size A'B'



$$\overline{CF'} = \frac{\overline{CS}}{2}$$

Real foyer

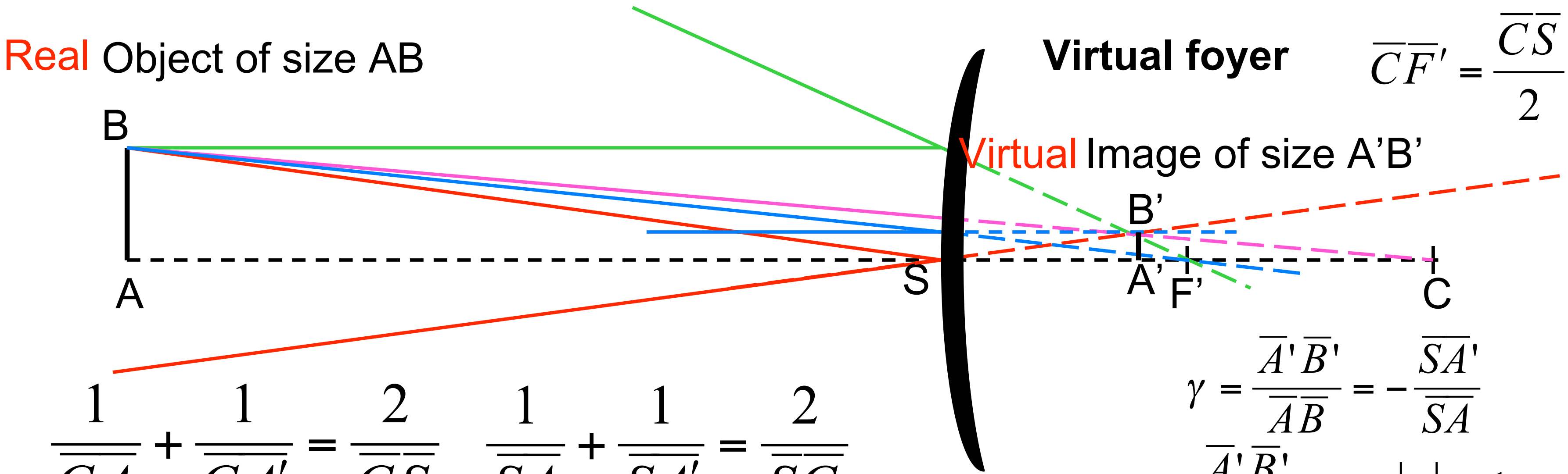
$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}$$

$$\frac{1}{\overline{CA}} + \frac{1}{\overline{CA'}} = \frac{2}{\overline{CS}} \qquad \frac{1}{\overline{SA}} + \frac{1}{\overline{SA'}} = \frac{2}{\overline{SC}}$$

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} < 0 \qquad |\gamma| < 1$$

- 4 rays **But 2 out of the 4 are sufficient !**
  - A ray going through the center of curvature is not deviated
  - A ray going through the summit S of the mirror is reflected symmetrically to the optical axis
  - An incident ray parallel to the optical axis goes through the image foyer
  - An incident ray going through the object foyer goes parallel to the optical axis

Ray tracing: case of a convexe mirror



$$\overline{CF'} = \frac{\overline{CS}}{2}$$

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}$$

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} > 0 \quad |\gamma| < 1$$

$$\frac{1}{\overline{CA}} + \frac{1}{\overline{CA'}} = \frac{2}{\overline{CS}} \qquad \frac{1}{\overline{SA}} + \frac{1}{\overline{SA'}} = \frac{2}{\overline{SC}}$$

- 4 rays **But 2 out of the 4 are sufficient !**
  - A ray going through the center of curvature is not deviated
  - A ray going through the summit S of the mirror is reflected symmetrically to the optical axis
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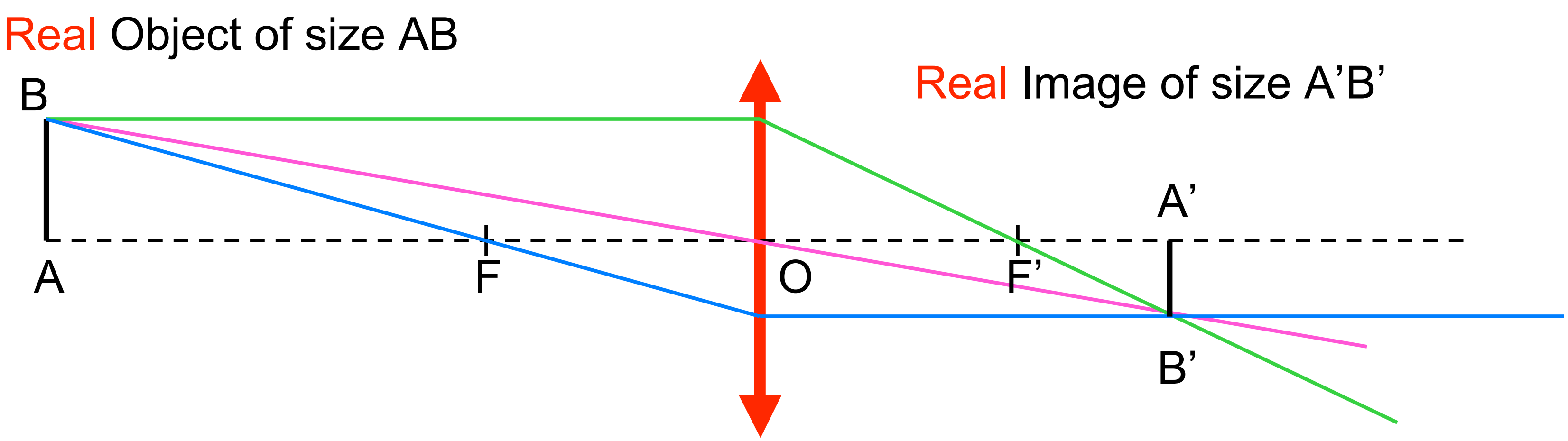
  - An incident ray going through the object foyer goes parallel to the optical axis

Ray tracing:
case of a convexe lens



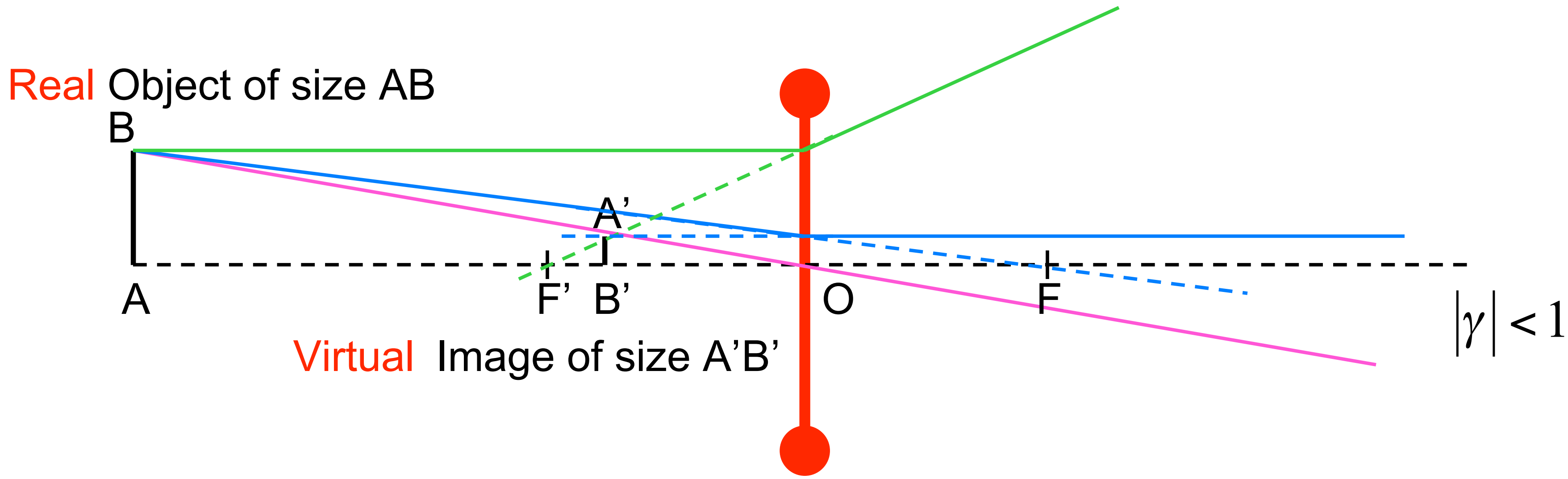
$$f' \geq 0 \qquad \gamma = \frac{\overline{A'}\,\overline{B'}}{\overline{A}\,\overline{B}} < 0 \qquad |\gamma| < 1$$

3 rays

- The ray going through the optical center is not deviated
- An incident ray parallel to the optical axis goes through the image foyer
- An incident ray going through the object foyer goes parallel to the optical axis

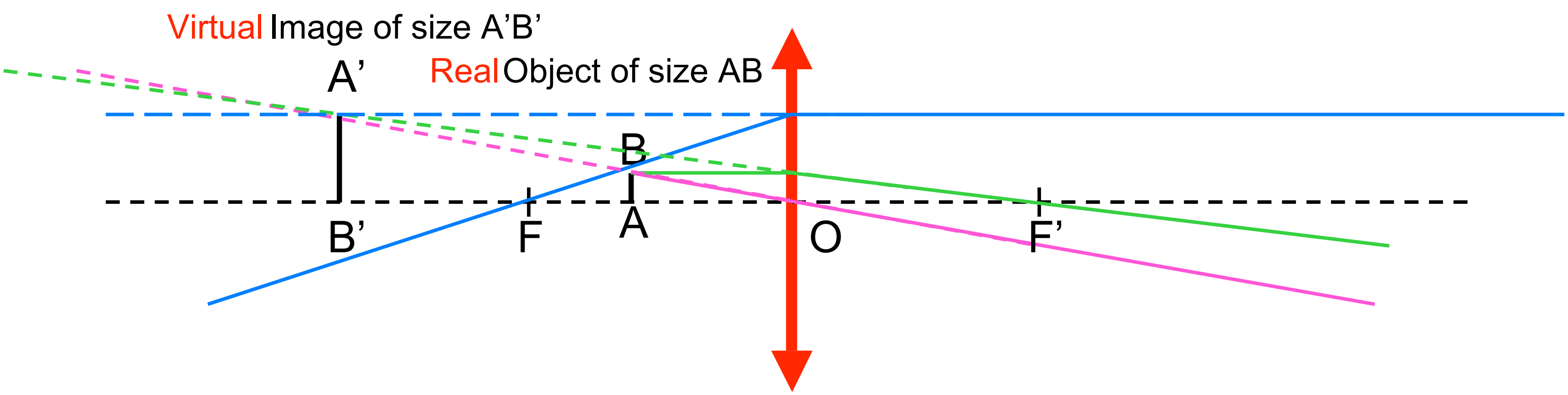
But 2 out of the 3 are sufficient !

Ray tracing: case of a concave lens



- 3 rays **(But 2 out of the 3 are sufficient !)**
  - The ray going through the optical center is not deviated
  - An incident ray parallel to the optical axis goes through the image foyer
  - An incident ray going through the object foyer goes parallel to the optical axis

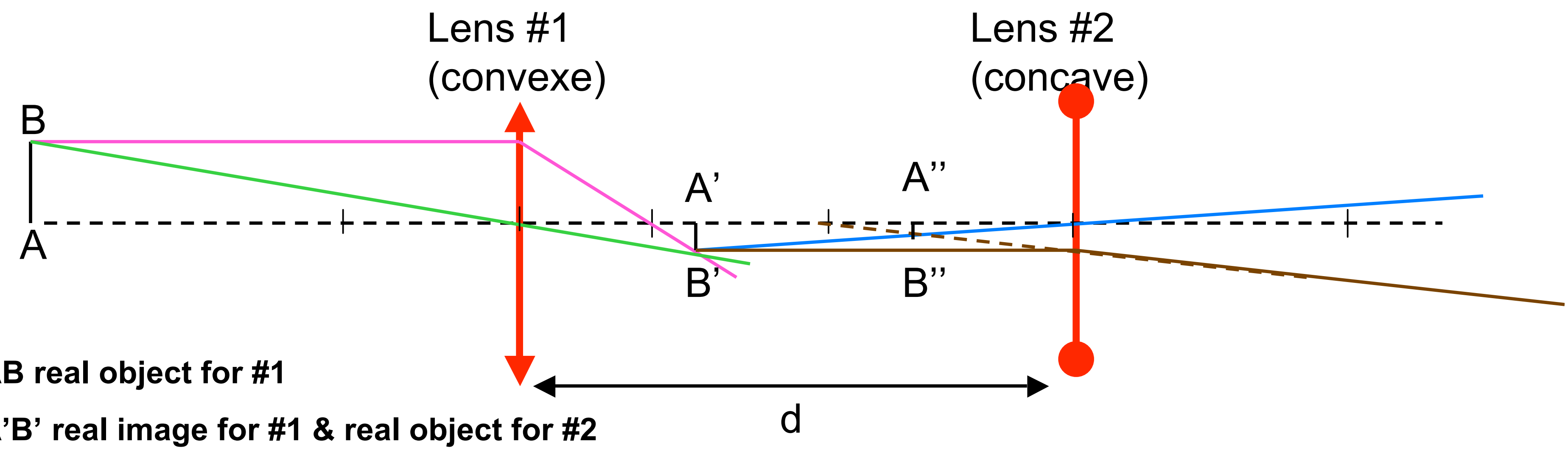
Another example



- **A convex lens does not necessarily yield a real image from a real object (idem for a concave lens and also for convex/concave mirrors)**

- **Object’s type (i.e. real or virtual) and image’s type is not only related to the curvature (convex or concave) of the lens (or mirror) but on the position of the object (resp. image) relatively to the object (resp. image) focal point**

Association of lenses



AB real object for #1

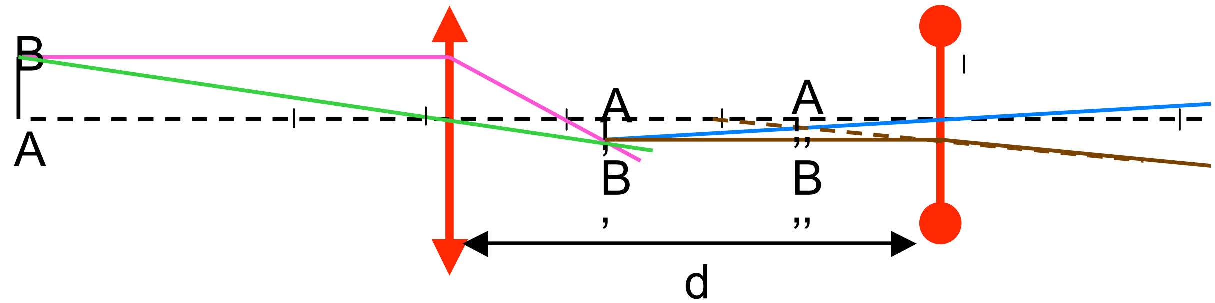
A'B' real image for #1 & real object for #2

A''B'' virtual image for #2

Formula for lens #1

Formula for lens #2

Association of lenses

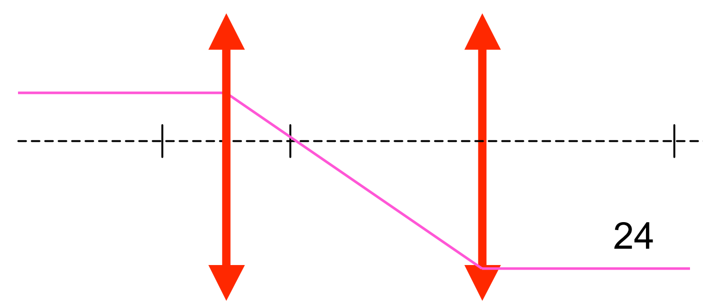


- Equivalent focal length?
- What if ?

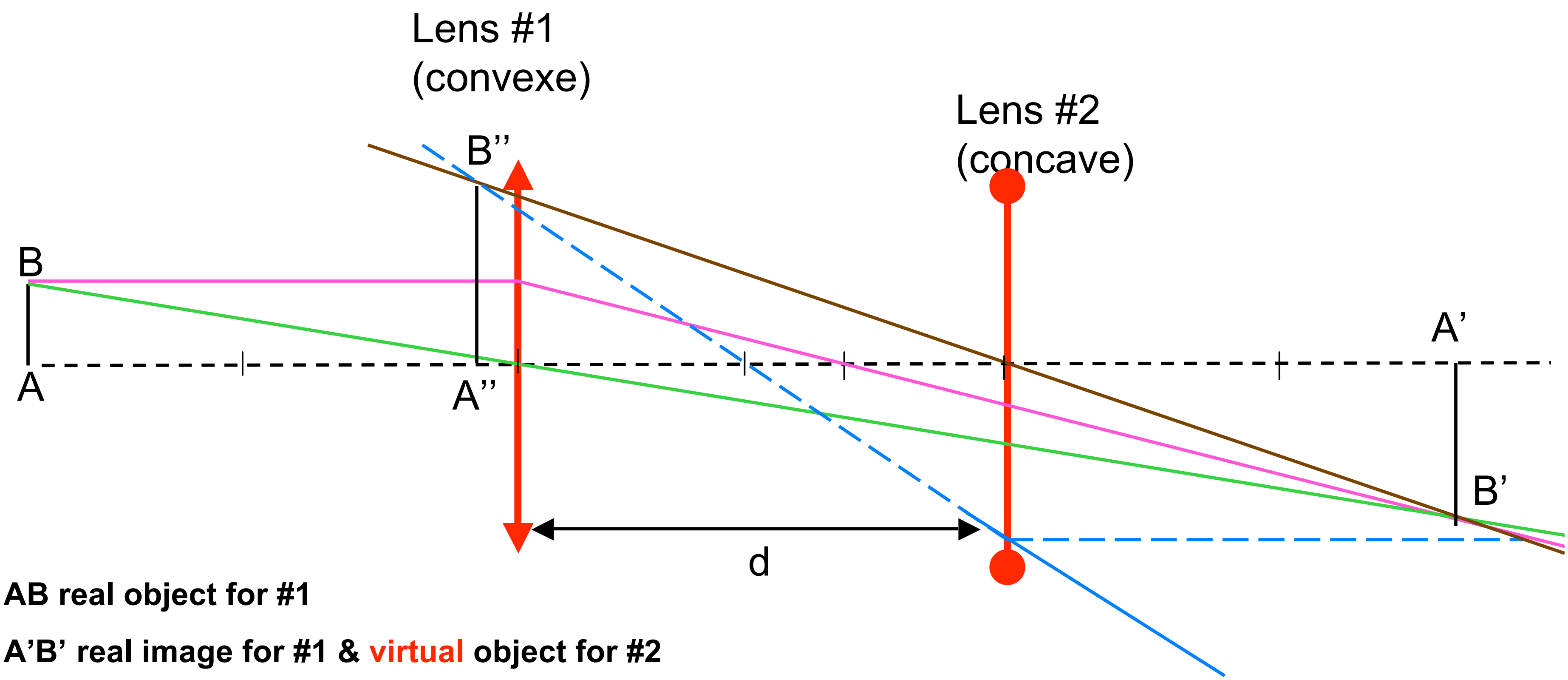
Association of lenses side by side = sum the dioptics

- What if ?

Afocal system



Association of lenses



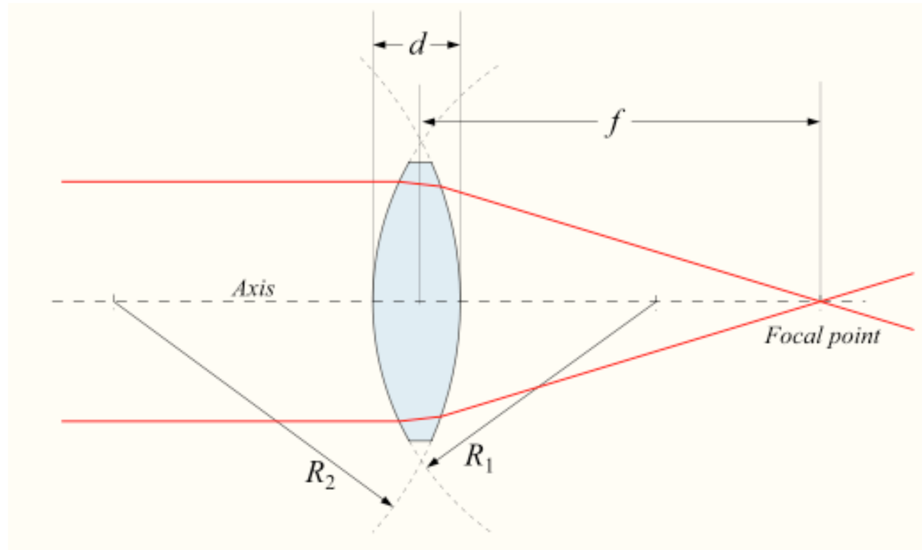
AB real object for #1

A'B' real image for #1 & virtual object for #2

A''B'' virtual image for #2

Advanced formulae

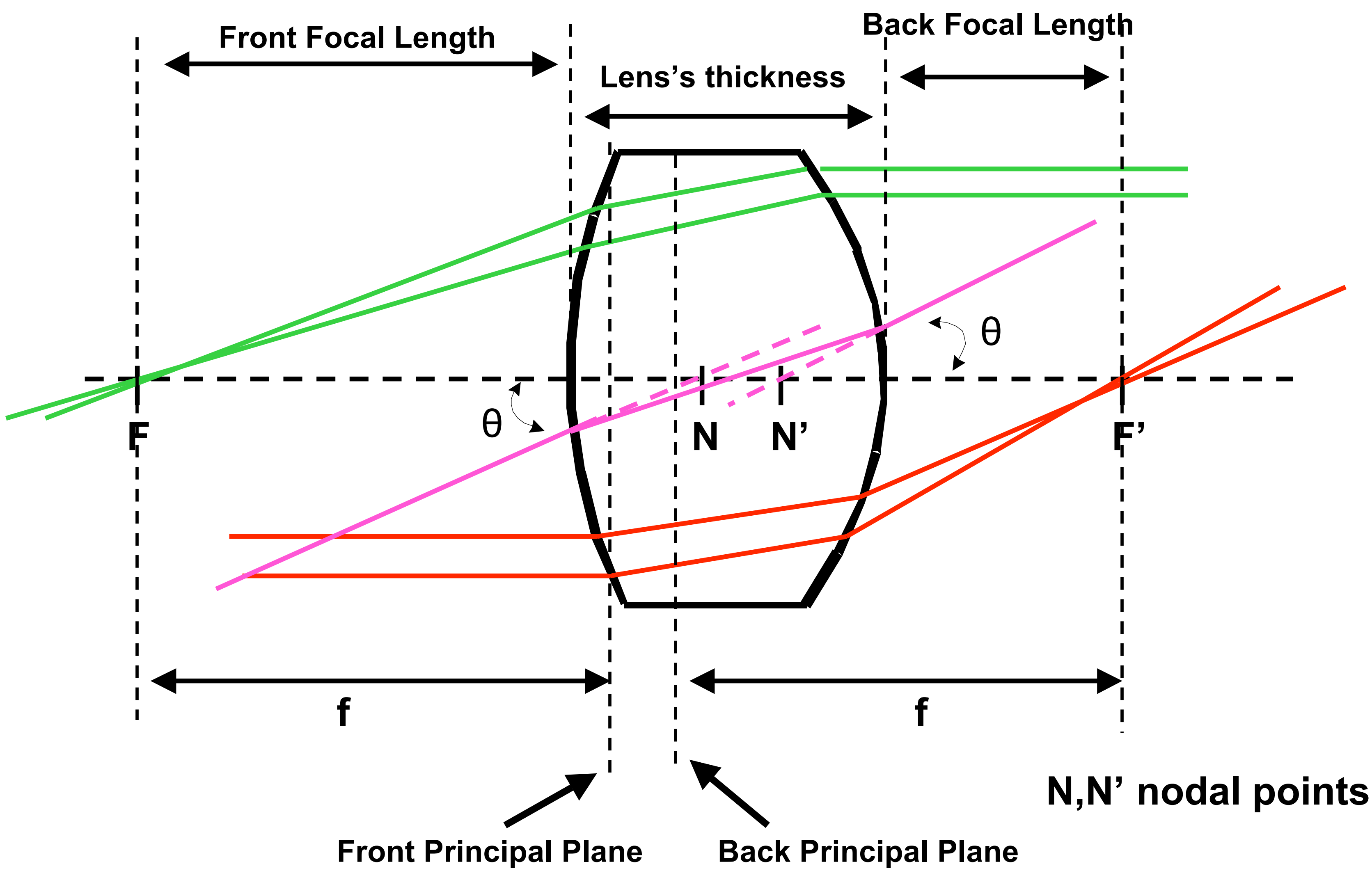
- Lensmaker's equation
- Thin lenses equations
- Thin lenses + Gauss' conditions
  - The focal lens is symmetrical f = f'



Positive (converging) lens

Convex
Concave
Concave
Convex

Thick lenses



Conclusion

- Gauss's condition for centered systems
  - For lenses: 3 possible rays
  - For mirrors: 4 possible rays
  - But 2 are always sufficient
- formula
  - Mirror $\frac{1}{\overline{CA}} + \frac{1}{\overline{CA'}} = \frac{2}{\overline{CS}}$

Concave mirror

Convex mirror

  - Lens

Convexe lens

Concave lens

Resources

- Books
  - Principles of Optics, 7th edition, Max Born & Emil Wolf
  - Optique, fondements et applications, 6th edition, 2000, José-Philippe Pérez, Dunod, Masson Sciences (French book)
  - The Manual of Photography, 9th edition, 2000, Focal Press
  - The principles of optics, Hardy and Perrin, 1932
- Web
  - Wikipedia (of course!)
  - http://www.esinsa.unice.fr/~vig/ESINSA1/Cours/Cours.html#chp3 (French)
  - http://grus.berkeley.edu/~jrg/Telescopes/node2.html (entire demonstration of formula for spherical mirrors)
- Comprehensive Software (do a lot more than ray tracing!)
  - Free software: Modas (free demo limited to 4 surfaces)
    - http://members.kabsi.at/i.krastev/modas/downloads.html
  - The Reference: Zemax (several min $2k/licence)
    - http://www.zemax.com/

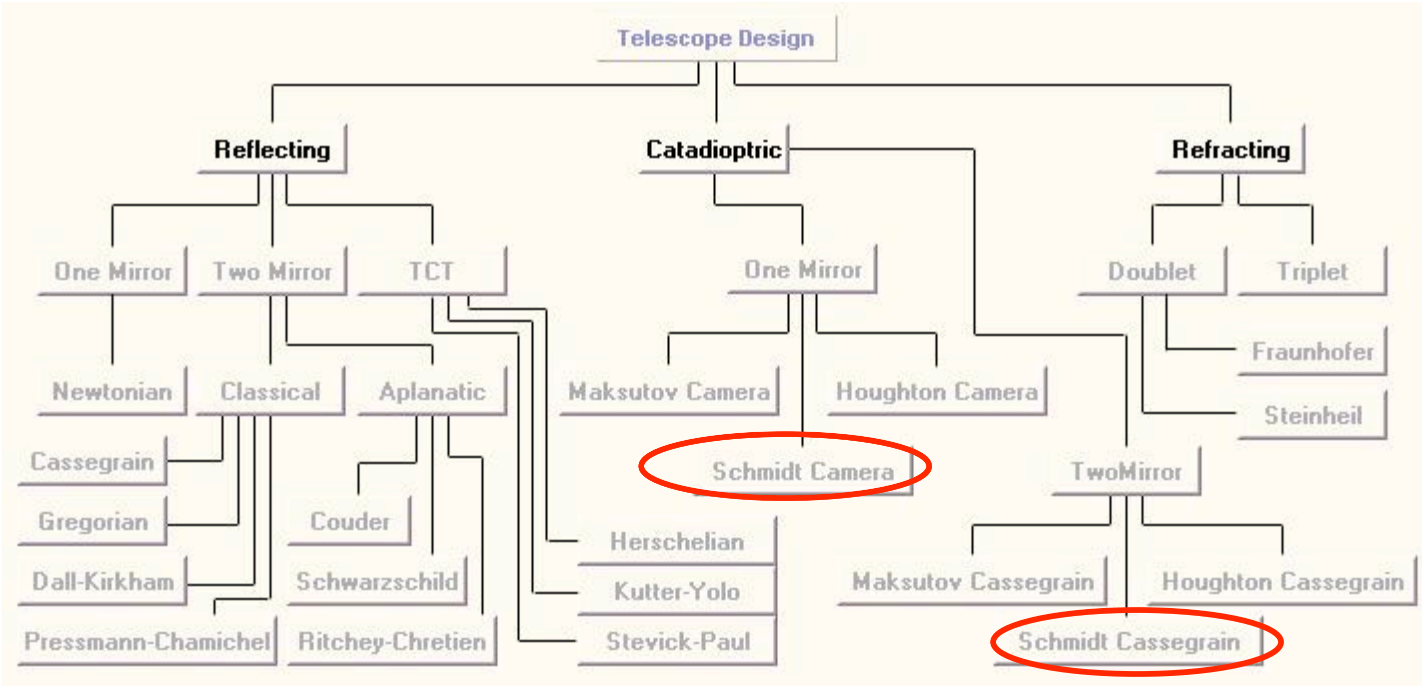
Some optical devices

- The eye
  - Short sighted
  - Long sighted
- Telescopes
  - Refractive
  - Reflective

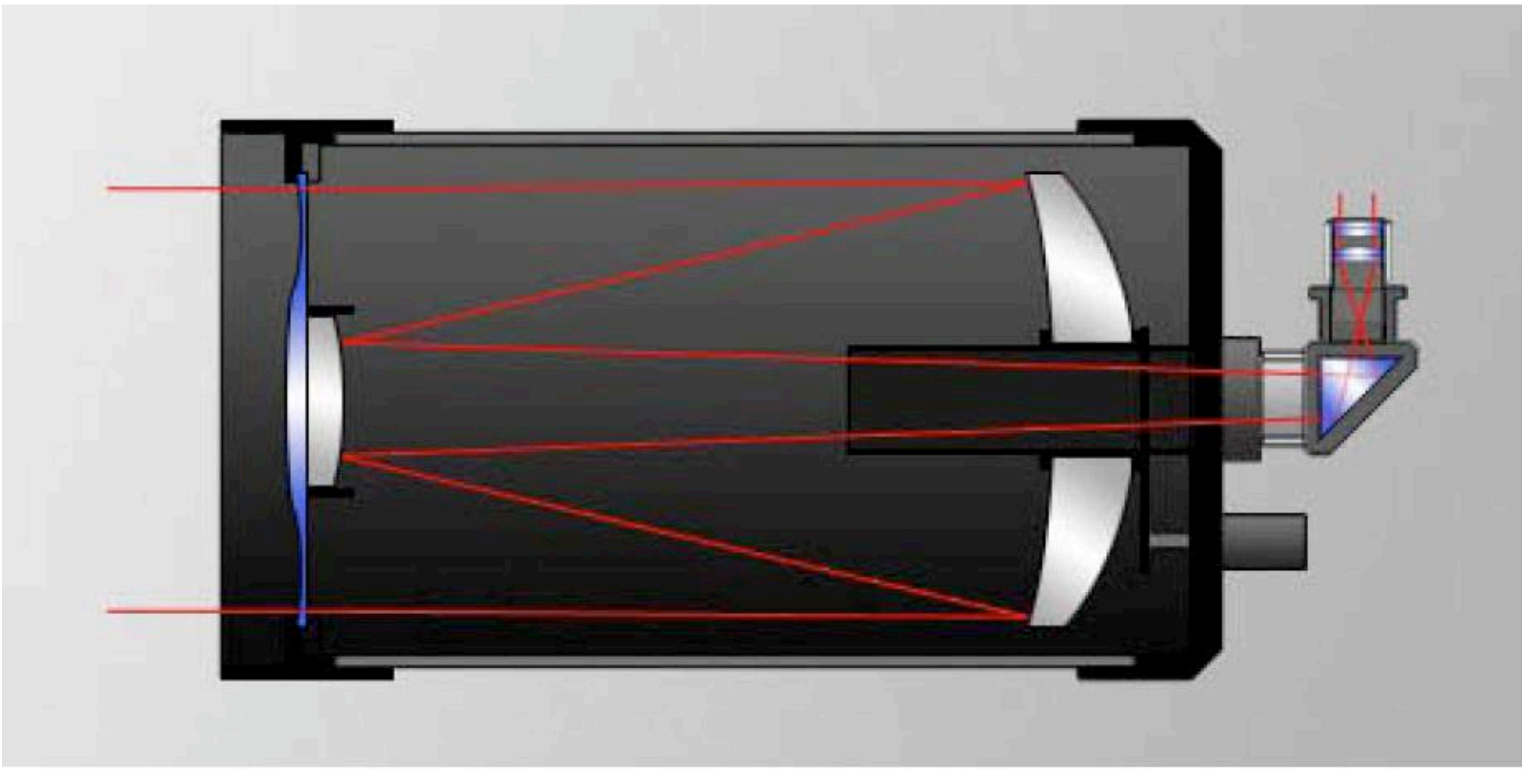
The eye

- Variation of the focal length of the eye
  - Focal length of punctum remotum = eye's focal length when object is @ infinity = ~2cm
  - Focal length of punctum proximum = eye's focal length when the object is at the minimum distance for which the image remains clear (typically ~10cm) is at ~1.85cm
- Long-sighed eye
  - When for an object placed at a finite distance, the image is behind the retina (not enough curvature of the eye's lens)
  - Need a convex corrective lens
- Short-sighed eye
  - When for an object at infinity, the image is before the retina (too much curvature of the eye's lens)
  - Need a concave corrective lens

Telescopes


Telescope Design
Reflecting
Catadioptric
Refracting
One Mirror
Two Mirror
TCT
One Mirror
Doublet
Triplet
Fraunhofer
Newtonian
Classical
Aplanatic
Maksutov Camera
Houghton Camera
Steinheil
Cassegrain
Schmidt Camera
TwoMirror
Gregorian
Couder
Herschelian
Dall-Kirkham
Schwarzschild
Kutter-Yolo
Maksutov Cassegrain
Houghton Cassegrain
Pressmann-Chamichel
Ritchey-Chretien
Stevick-Paul
Schmidt Cassegrain

Typical design of a Schmidt Cassegrain

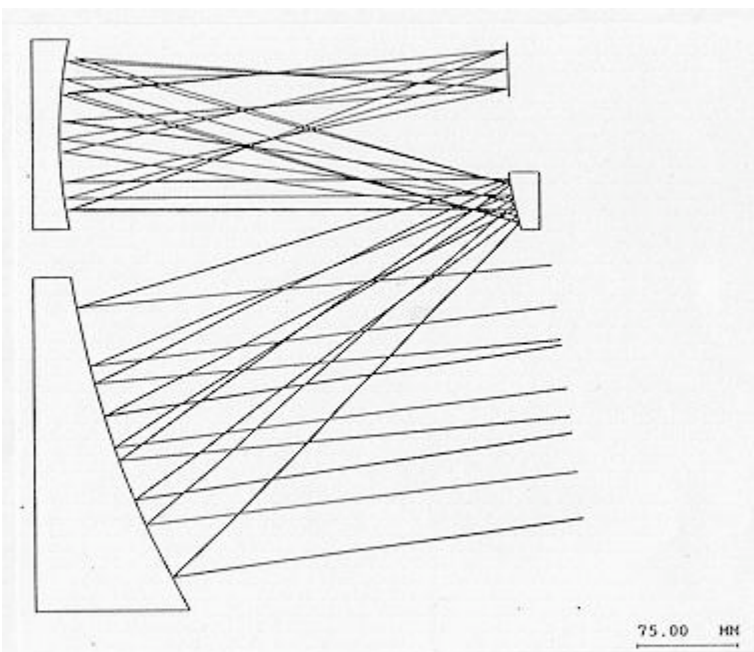


Concave primary + convex secondary = Cassegrain
Cassegrain + Schmidt corrector = Schmidt Cassegrain
Hyperbolic primary and secondary = Ritchey-Chrétien

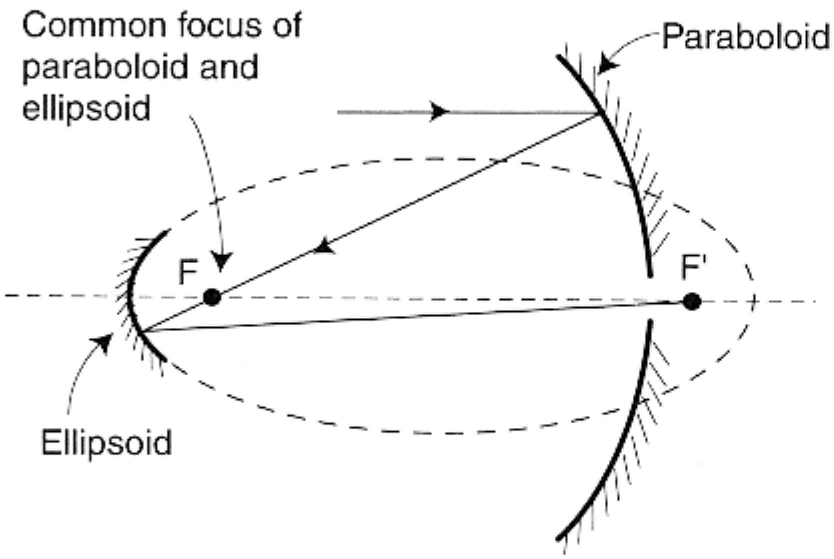
TMA
(Three Mirrors Anastigmatic)

- 3 curved mirrors = more degree of freedom
  - Primary
  - Secondary
  - Tertiary
- One fold mirror (usually)
- Off axis systems

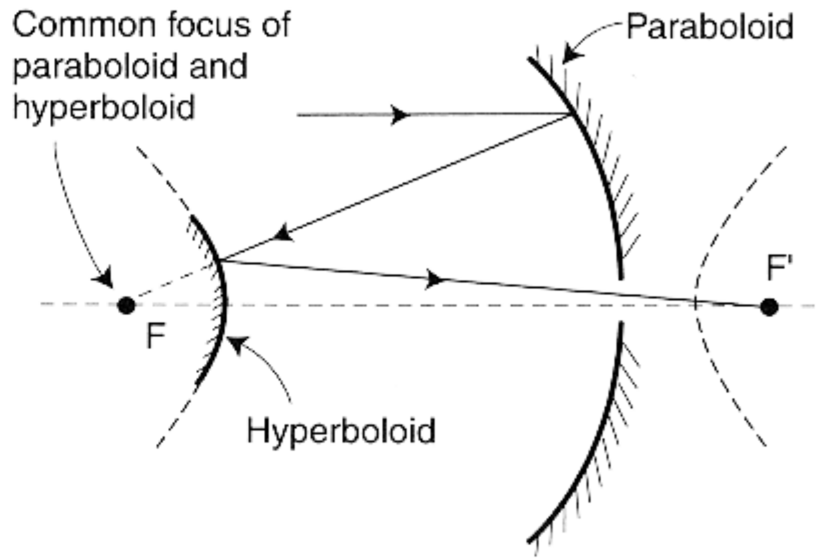
- Pros
  - Better correction of aberrations (more dof)
  - Very compact system
- Cons
  - Very complex (hard to maintain position under temperature variations)…
  - … therefore more $$$
  - Centered system are blind in the middle of the focal plane



Curvature of mirrors
Manufacturing issues



Gregorian telescope



Cassegrain telescope

Why always/often spherical lenses and mirrors?
Why not paraboloic/ellipsoic/hyperboloic shapes?

Answer: hard/expensive to manufacture.
Common accuracy is λ/20